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A note on the role of zero-point energy in evolutionary cosmology

N. KUMAR†
Physics Department, University of British Columbia, Vancouver, B.C., Canada
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Abstract. Following a suggestion originally due to McCrea, a physical vaccum is regarded as the ground state of a certain quantized field obeying Bose statistics. Under certain assumptions regarding the physical nature of the zero-point energy (assumed to be essentially positive) associated with the Bose field, the latter is found to result in an effectively negative kinematic pressure p having the functional dependence of the form $p \propto G^{-4}$, where G is the curvature of space. This leads naturally to an oscillating isotropic model of the universe without singularity, of the type treated in detail by Pachner. In effect, the purely mathematical artifice of assuming negative pressure is given a physical basis in terms of the zero-point energy considerations.

We consider here a general cosmological model which is described from purely kinematical considerations by the Robertson–Walker line element, as given by

$$ds^2 = c^2\, dt^2 - \frac{R(t)/R_0}{(1+kr^2/4R_0{}^2)^2}(dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2) \tag{1}$$

where the curvature index k can take on values $\pm 1, 0$, and the expansion function $R(t)$ is a positive function of the cosmic time t. R_0 is a constant parameter having the dimension of length. r, θ, ϕ are the usual spherical polar coordinates of the fundamental observer at the epoch t. c is the velocity of light *in vacuo*. The distance between any two neighbouring points, comoving with the coordinate mesh, is a constant times $R(t)$. This interpretation of the time-dependent expansion function is essential to the present treatment. It may well be emphasized that this expansion is of a global nature inasmuch as it leaves unaltered all the fundamental lengths dynamically associated with the atomic (local) sub-systems. Thus, for instance, the atomic Bohr radius continues to provide, operationally, an unchanging measure of length. On the other hand, and this is the adiabatic hypothesis essential to the present treatment, all the normal modes, spanning the cosmic space sections, have their wavelengths increased adiabatically as a result of the Hubble expansion. This, of course, gives rise to the well-known cosmological red shift. The adiabaticity follows directly from the kinematic nature of the Robertson–Walker line element assumed. In the present note we consider the effect of this universal expansion on the modes of zero-point vacuum fluctuations associated with a quantized field obeying Bose statistics, and the dynamical consequences thereof.

Substituting the Robertson–Walker metric (1), with positive curvature ($k = +1$), in the field equations of general relativity

$$R_{ij} - \tfrac{1}{2}g_{ij}R = -\kappa T_{ij} \tag{2}$$

with $\kappa = 8\pi\gamma/c^2$ (in the usual notation), we get the well-known system of two ordinary differential equations (Pachner 1965):

$$\frac{2\ddot{R}}{c^2R} + \left(\frac{\dot{R}}{cR}\right)^2 + \left(\frac{1}{R}\right)^2 = -\kappa p \tag{3}$$

$$\left(\frac{\dot{R}}{cR}\right)^2 + \left(\frac{1}{R}\right)^2 = \kappa\rho \tag{4}$$

† On leave of absence from the National Chemical Laboratory, Poona 8, India.

where overhead dots denote 'differentiation' with respect to the cosmic time t. ρ and p are, respectively, the observable mass density (smoothed-out or relative density) and pressure, assumed to be spatially uniform and isotropic for reasons of symmetry. In terms of the diagonal components of the energy–stress tensor $T_i{}^j$, we have

$$\begin{aligned} T_4{}^4 &= \rho(R) \\ T_1{}^1 = T_2{}^2 = T_3{}^3 &= -\frac{p(R)}{c^2} \\ T_i{}^j &= 0,\, i \neq j. \end{aligned} \tag{5}$$

In (5) the argument $R(t)$ has been shown explicitly to indicate a possible parametric dependence on $R(t)$ and, therefore, indirectly on t. A specification of the functional dependence $p(R)$ determines the solution of the problem to within the initial conditions. Pachner (1965) has analysed mathematically general properties of the solutions of the equations (3) and (4) for a class of functional dependences of the type $p/c^2 = -aR^{-n}$, for integral n. The particular case of $n = 4$ was treated in detail, and the condition for an oscillating isotropic universe without singularity was found to be $a > 0$. The latter implied negative pressure and, in turn, creation of matter.

In what follows we show that the functional dependence $p/c^2 = -aR^{-4}$ as well as the negative sign for pressure (i.e. $a > 0$) follows as a necessary consequence from certain considerations attending the zero-point energy.

Following a suggestion originally due to McCrea (1951), we regard the space (mostly vacuum) as representing the quantum-mechanical ground state of a certain boson field. The nature of the boson field is, however, quite arbitrary, except that it must be capable of long-range quasi-static interactions so as to be of any cosmic significance. This limits the choice to only such fields as are associated with the zero-mass particles (e.g. graviton, having spin 2). The photon field is probably eliminated because of the overall charge neutrality of the universe. The assumption of ground state is justified (except perhaps for the first few seconds of the beginning of the expansion) by the fact that the interstellar space is essentially at the absolute zero of temperature. Indeed the 3 °K background radiation density ($\sim 10^{-34}$ g cm^{-3}) constitutes a negligibly small portion of the visible matter density ($\sim 10^{-30}$ g cm^{-3}). Thus, one has to consider only the zero-point energy of the boson field which is taken to be the graviton field in the present case.

The mass equivalent of the zero-point energy density ρ_{zp} is given by the usual relation

$$\rho_{zp} = \frac{1}{2(2\pi)^3} \sum_j \int_0^{k_c(t) = 2\pi/\lambda_c(t)} \frac{\hbar w_j(k)\, d^3\mathbf{k}}{c^2} \tag{6}$$

where $\hbar w_j(k)$ is the mode-branch energy, j the branch index and $\mathbf{k}$ the propagation vector. $\hbar$ and c are, respectively, Planck's constant and velocity of light *in vacuo*. $k_c(t)$ and $\lambda_c(t)$ are, respectively, the cut-off wave vector magnitude and the corresponding wavelength. Their possible parametric time dependence has been indicated explicitly in accordance with the adiabatic hypothesis. Thus we have

$$k_c(t) = \frac{2\pi}{\lambda_c(t)} \propto R(t)^{-1}, \qquad \text{or} \qquad k_c(t) = k_c(0)\frac{R_0}{R(t)} \tag{7}$$

and also

$$\frac{1}{\lambda_c(t)}\frac{\mathrm{d}\lambda_c(t)}{\mathrm{d}t} = \frac{1}{R(t)}\frac{\mathrm{d}R(t)}{\mathrm{d}t} \equiv H(t) \text{ (the Hubble 'constant' at the epoch } t). \tag{8}$$

To derive an expression for the pressure p_{zp} associated with the zero-point energy density, we follow a procedure essentially similar to that given by Casimir (1948) for calculating the non-retarded interaction energy between two neutral macroscopic bodies. Accordingly, we apply the energy balance condition to an element of space section with boundaries comoving with the coordinate mesh. We obtain after some simple reduction

$$\frac{\partial \rho_{zp}}{\partial t} + 3\rho_{zp}H + \frac{3}{c^2}p_{zp}H = 0. \tag{9}$$

Equation (9) is evidently a continuity equation for the zero-point energy content of space. The first term is the parametric rate of change of the zero-point mass density (the partial derivative is to be understood in this sense) and represents the cosmological red shift (an expansion effect). The second term is the streaming term, while the third term is the mass equivalent of the adiabatic work done by the pressure p_{zp} in the expansion process.

Combining (6), (7), (8) and (9) and using the well-known dispersion relation $w_j(k) = ck$ as given by Dirac, we obtain

$$\frac{p_{zp}}{c^2} = \frac{\alpha\hbar}{32\pi^2 c k_c^4(0)}\left\{\frac{R_0}{R(t)}\right\}^4 \tag{10}$$

where α is the number of branches assumed to be degenerate. Now we make the physically plausible assumptiou that the zero-point energy as such is not observable. It manifests itself only in virtue of the associated pressure p_{pz}. Thus, we have the picture in which the zero-point modes are being cosmologically red-shifted owing to expansion, and the zero-point energy so depleted manifests itself as the adiabatic work done by the pressure p_{zp}. If, however, we retain the classical notion of conservation of overall energy, the mass equivalent of the adiabatic work must necessarily reappear as accession to the observable mass density. Mathematically, this continual accretion of mass can be incorporated in equations (3) and (4) by postulating an additional pressure equal and opposite to p_{zp}. Neglecting any other component of pressure, we have

$$\begin{aligned}\frac{p}{c^2} = \frac{-p_{zp}}{c^2} &= -\frac{\alpha\hbar R_0^4}{32\pi^2 c k_c^4(0)} R(t)^{-4} \\ &= -aR(t)^{-4} \end{aligned} \tag{11}$$

or

$$T_1^1 = T_2^2 = T_3^3 = aR(t)^4$$

where

$$a = \frac{\alpha\hbar R_0^4}{32\pi^2 c k_c^4(0)}. \tag{12}$$

This is exactly the same as that obtained by Pachner (1965). As shown by him, the expansion function $R(t)$ in this case is periodic in time and varies between two extremes in the fashion of a curtate cycloid. Also, defining

$$\max R = R_0, \qquad \min R = \epsilon R_0, \qquad 0 \leqslant \epsilon \leqslant 1 \tag{13a}$$

a was found to be given by

$$\kappa a = \epsilon R_0^2. \tag{13b}$$

From (7) and (13) we have

$$k_c(0) \leqslant k_c(t) \leqslant \frac{k_c(0)}{\epsilon}. \tag{14}$$

R_0 is given in terms of the time interval T_0 elapsed between the beginning of the expansion and its maximum value as

$$T_0 = (1+\epsilon)\tfrac{1}{2}\pi\frac{R_0}{c}. \tag{15}$$

Assuming Pachner's estimates for ϵ (the darkness of the sky) and T_0, namely

$$\epsilon \sim 10^{-21} \text{ and } T_0 \sim 10^{17}\,\text{s}$$

and using (12), (13) and (14), we obtain for the limits of the cut-off parameter $k_c(t)$ (orders of magnitude only)

$$1 \text{ cm}^{-1} \leqslant k_c(t) \leqslant 10^{21} \text{ cm}^{-1}.$$

If we identify the present epoch with $t = T_0$, as done by Pachner, the present value of the cut-off frequency $w_c(0) = ck_c(0)$ turns out to be

$$w_c(0) \sim 10^{10} \text{ s}^{-1}$$

which is a relatively low value.

The smallness of the effective graviton cut-off frequency can, perhaps, be attributed to the extremely small probability of emission or absorption of the graviton quanta. (The characteristic time is of the order of 10^{40} s while that for the case of a photon is only 10^{-12} s.) Thus, the high-frequency components of graviton modes are extremely unlikely to be excited, if at all (Bergmann 1964).

The above analysis purports to show that the quantal zero-point energy effects may be significant in cosmological problems. Also, the particular case studied by Pachner ($n = 4$, $a > 0$) is of more than purely mathematical significance.

A further point of interest that emerges from the foregoing treatment is the following. The 'creation' (or rather the re-creation) process envisaged here involves bosons (i.e. the zero-point gravitons). As is well known, because of the degenerate nature of Bose statistics, quite generally, the cross section for such a process must increase monotonically with the increasing boson density, already present. This would physically imply that the 'creation' process is enhanced in the vicinity of massive gravitating bodies (i.e. regions of higher graviton density). This may have relevance to the modified creation field theories inasmuch as the latter predict the creation process to be localized in the materially dense regions of the universe. Further analysis in this direction is in progress.

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